AlphaZero-Inspired General Board Game Learning and Playing

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***Abstract*—Recently, the seminal algorithms AlphaGo and AlphaZero have started a new era in game learning and deep reinforcement learning. While the achievements of AlphaGo and AlphaZero – playing Go and other complex games at super human level – are truly impressive, these architectures have the drawback that they are very complex and require high computational resources. Many researchers are looking for methods that are similar to AlphaZero, but have lower computational demands and are thus more easily reproducible. In this paper, we pick an important element of AlphaZero – the Monte Carlo Tree Search (MCTS) planning stage – and combine it with reinforcement learning (RL) agents. We wrap MCTS for the first time around RL n-tuple networks to create versatile agents that keep at the same time the computational demands low. We apply this new architecture to several complex games (Othello, ConnectFour, Rubik's Cube) and show the advantages achieved with this AlphaZero-inspired MCTS wrapper. In particular, we present results that this AlphaZero-inspired agent is the first one trained on standard hardware (no GPU or TPU) to beat the very strong Othello program Edax up to and including level 7 (where most other algorithms could only defeat Edax up to level 2).**

I. Introduction

A. Motivation

In computer science, game learning and game playing are interesting test beds for strategic decision making done by computers. Games usually have large state spaces, and they often require complex pattern recognition and strategic planning capabilities to decide which move is the best in a certain situation. If an algorithm is able to learn a game (or, even better, a variety of different games) just by self-play, given no other knowledge than the game rules, it is likely to perform also well on other problems of strategic decision making.

With their seminal papers on AlphaGo [1], AlphaGo Zero [2] and AlphaZero [3], Silver et al. opened a new door in game learning by presenting self-learning algorithms for the game of Go (which was considered to be unattainable for computers prior to theses publications). As we all know, all these algorithms were able to beat the human Go world champion Lee Sedol.

However, the full algorithms AlphaGo or AlphaZero require huge computational resources in order to learn how to play the game of Go at world-master level. It is the purpose of this work to investigate whether some of the important elements of AlphaZero can already reach decent advances in game learning with much smaller computational efforts. For this purpose, we study several games – namely Othello, ConnectFour and Rubik's Cube – that have a lower complexity than Go yet are not easy to master for both humans and game learning algorithms. The goal is to deliver not only agents with average game playing strength but agents that play near-perfect on that games. We will show that this can be achieved for Othello and ConnectFour and, to some extent, also for Rubik's Cube.

In this work, we pick an element of AlphaZero (here: the MCTS planning stage) and combine it with RL agents. Here, we wrap MCTS for the first time around TD-n-tuple networks, but the same technique could be applied to all types of RL agents. We apply this new architecture with low computational demands to several complex games and show that great advantages are achieved with this AlphaZero-inspired MCTS wrapper.

B. Related work

The seminal papers of Silver et al. on AlphaGo and AlphaZero [1], [3] have stirred the interest of many researchers to achieve similar things with smaller hardware requirements and/or fewer training cycles. Thakoor et al. [4] already provided 2017 a general AlphaZero implementation in Python with less computational demands than the original. But even their architecture requires 3 days of training on a specialized cloud computing service (Google Compute Engine with GPU support). Several works of Wang et al. [5], [6], [7] focus on different aspects of the AlphaZero architecture: alternative loss functions, hyperparameter tuning and warm-start enhancements. They test these aspects on smaller games like 6x6 Othello or 5x5 ConnectFour. Young et al. [8] report on an AlphaZero implementation applied to ConnectFour. Here, training took between 21 and 77 hours of GPU time. The work of Chang et al. [9] covered several AlphaZero improvements applied to 6x6 Othello.

Recently in 2021, Norelli and Panconesi [10] came up with a very interesting paper that is close to our work: They pursue as well the goal to set up an AlphaZero-inspired algorithm at much lower cost than the original AlphaZero [3]. The agent in [10] is trained solely by self-play, is able to play 8x8 Othello and defeat the strong Othello program Edax [11] up to level 10. Although much less computationally demanding than the original AlphaZero [3], their training time took roughly one month on Colaboratory, a free Google cloud computing service offering GPUs and TPUs.

Apart from Norelli and Panconesi [10], there are only few works on Othello game learning that actually benchmark against Edax: Liskowski et al. [12] presented in 2018 an agent obtained by training a convolutional neural network (CNN) with the help of a database of expert moves. Their agent could defeat Edax up to and including level 2.

Our work presented here is based on an earlier Bachelor thesis [13] published in 2020 (but only in German); it presents an n-tuple RL agent trained in 1.8 hours on standard hardware (no GPU) that can defeat Edax up to and including level 7. See Sec. V (Discussion) for further comparison between Norelli and Panconesi [10] and our work.

N-tuple networks, which are an important building block of our approach, have shown to work well in many games, e.g., in ConnectFour [14], [15], Othello [16], EinStein würfelt nicht [17], 2048 [18], SZ-Tetris [19] etc. Other function approximation networks (deep neural networks or other) could be used as well in AlphaZero-inspired reinforcement learning, but n-tuple networks have the advantage that they can be trained very fast on off-the-shelf hardware.

The algorithm presented in this paper is implemented in the General Board Game (GBG) learning and playing framework [20], [21], which was developed for education and research in AI. GBG allows applying the new algorithm easily to a variety of games. GBG's open source code is available on GitHub[1].

A work related to GBG [20], [21] is the general game system Ludii [22]. Ludii is an efficient general game system based on a ludeme library implemented in Java, allowing to play as well as to generate a large variety of strategy games. Currently, all AI agents implemented in Ludii are tree-based agents (MCTS variants or AlphaBeta). GBG, on the other hand, offers the possibility to train RL-based algorithms on several games.

The main contributions of this paper are as follows: (i) it shows for the first time – to the best of our knowledge – an AlphaZero-like coupling between n-tuple networks and MCTS planning; (ii) an AlphaZero-inspired solution with largely reduced computational requirements; (iii) very good results on Othello, ConnectFour and 2x2x2 Rubik's Cube.

The rest of this paper is organized as follows: Sec. II details the algorithmic building blocks and methods of our approach. Sec. III describes the experimental setup, the games and the evaluation methods. Sec. IV shows the results on the three games: quality achieved, interpretation, computation times. Sec. V discusses the results in comparison with other research and Sec. VI concludes.

II. Algorithms and Methods

A. Algorithm Overview

The most important task of a game-playing agent is, given an observation or game state s_t at time t, to propose a good next action a_t from the set of actions available in s_t (Fig. 1). TD-learning uses the value function $V(s_t)$, which is the expected sum of future rewards when being in state s_t.

It is the task of the agent to learn the value function $V(s)$ from experience (by interacting with the environment). In order to do so, it usually performs multiple self-play training episodes until a certain training budget is exhausted or a certain game-playing strength is reached.

Our base RL algorithm TD-FARL is described in detail in [23], [24] and is partly inspired by Jaskowski et al. [25], van der Ree et al. [26] and partly from our own experience with RL-n-tuple training. The key elements of the new RL-logic – as opposed to our previous RL algorithms [14], [27] – are N-tuple systems, temporal coherence learning (TCL) [28] and final adaptation RL (FARL) [23], [24]. The last element (FARL) was necessary to create an algorithm that works successfully in various N-player games with arbitrary N [24], [26].

Despite being successful on a variety of games [23], [24], this base algorithm shares one disadvantage with other deep learning algorithms that are only value-based: they base their decision on the value of the current state-action pairs. They have no planning component, no what-if scenarios to think about further consequences, like possible counter-actions of the other player(s), further own actions and so on.

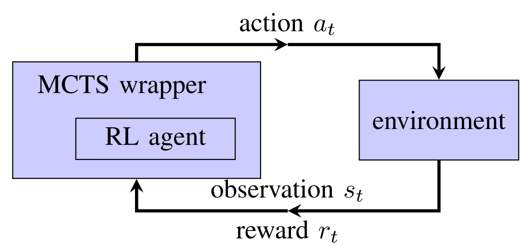


Figure 1. Reinforcement learning with MCTS wrapper: The RL agent with MCTS wrapper observes a certain state s_t and reward r_t from the game environment and predicts the next action a_t.

This is where AlphaZero's MCTS-trick comes into play: Silver et al. [1], [2] combine a deep learning RL agent with an MCTS wrapper (Fig. 1) to introduce such a planning component. They do this throughout the whole training procedure, which is better for the overall performance but which is also very computationally demanding. In this work, we take a simpler approach: we first train our RL agent, a TD n-tuple network, and then use the MCTS wrapping only at prediction time.

B. MCTS Wrapper

In theory, applying the Minimax algorithm to assess the entire game tree leads to an optimal game strategy for deterministic 2-player games with perfect information [29]. However, such brute-force-like algorithms may quickly reach their limits in practice for very large game trees - even optimizations like alpha-beta pruning can only counteract this to a limited extent.

[1]https://github.com/WolfgangKonen/GBG

Algorithm 1. MCTSITERATION: This recursive algorithm is applicable to 1- or 2-player games. It performs a single iteration of a Monte Carlo tree search, starting from root node R carrying state s.

```
1:  function MCTSITERATION(NODE R)
2:      κ = (−1)^(N−1)                         ▷ N: number of players
3:      if ISGAMEOVER(s) then
4:          return κ ∗ FINALGAMESCORE(s)
5:      if R.EXPANDED = FALSE then
6:          (V, p) ← f(s)                     ▷ f: approximator network
7:          P(s, ·) ← p                ▷ prior probabilities given by f
8:          R.EXPANDED ← TRUE
9:          return κ ∗ V
10:
11:     (a, C) ← SELECTCHILD(R)            ▷ use Eq. (1) to select
12:                                        ▷ action a and child C
13:     V_child ← MCTSITERATION(C)
14:     Q(s, a) ← Q(s, a) + V_child
15:     N(s, a) ← N(s, a) + 1
16:
17:     return κ ∗ V_child
```

Assuming the UCT variant of the Monte Carlo tree search, the probability of predicting an optimal move converges to 100% in the limit of an infinite number of iterations [30]. If we limit the iterations to a fixed size, we approximate optimality only but have a fixed runtime.

Therefore, with an MCTS, promising results can be expected under reasonable computational requirements, given the number of MCTS iterations is correctly balanced.

Another advantage of MCTS is that it can be interrupted prematurely and still deliver valuable results as a so-called anytime algorithm [31]. This property is beneficial when using hardware with limited computing power, especially with games that impose a move-based time limit. Thus, even if the available computing time is not sufficient to carry out all planned MCTS iterations, it is still possible to stop the search after any iteration and predict the most promising move at that time.

The phases of MCTS usually consist of four consecutive steps: Selection, expansion, simulation, and backpropagation [31]. The following child selection policy, which is the one used by Silver et al. [2] in AlphaGo Zero, is also the one that we have implemented in our MCTS wrapper for the same purpose:

$$a_t = \arg\max_{a \in A(s_t)} \left(\frac{Q(s_t, a)}{N(s_t, a)} + U(s_t, a) \right) \quad (1)$$

$$U(s, a) = c_{puct} P(s, a) \frac{\sqrt{\varepsilon + \sum_{b \in A(s)} N(s, b)}}{1 + N(s, a)} \quad (2)$$

Here, $Q(s, a)$ is the accumulator for all backpropagated values (as detailed in Algorithm 1 below) that arrive along with branch a of node R that carries state s. Likewise, $N(s, a)$ is the visit counter and $P(s, a)$ the prior probability. $A(s)$ is the set of actions available in state s. ε is a small positive constant for the special case $\sum_b N(s, b) = 0$: It guarantees that in this special case the maximum of $U(s, a)$ is given by the maximum of $P(s, a)$. The prior probabilities $P(s, a)$ are obtained by sending the values of all available actions $a \in A(s)$ through a softmax function.

According to Silver et al. [2], the above child selection policy is a variant of the PUCB ("Predictor + UCB") algorithm presented by Rosin [32]. Furthermore, the latter is a modification of the bandit algorithm UCB1, extending it with the behavior to also consider the recommendations of a predictor. UCB1 is also the basis of the previously mentioned algorithm UCT (UCB applied to trees) by Kocsis and Szepesvári [30].

Our implementation of a Monte Carlo tree search iteration is illustrated in Algorithm 1. It performs a single iteration of the Monte Carlo tree search for a given node. The numerical return value approximates how valuable it is to choose an action that leads to this node. Since this assessment corresponds to the view of the previous player, for 2-player games, the algorithm negates the returned values ($\kappa = -1$).

If the node represents a game-over state, then the consequence of choosing this node is known and does not need to be approximated. In this case, the final game score gives the evaluation value to propagate back.

Reaching a non-expanded node is also a termination condition. In this case, the approximator function f (usually the wrapped RL agent of Fig. 1) approximates the value V of the corresponding node together with its action probabilities $\mathbf{p}$ (line 6). Afterward, the node is marked as expanded, and its approximated value is propagated back.

SELECTCHILD is used to select a child node based on the PUCB variant of Eq. (1) if no previous termination condition occurred. To determine the selected child node's value, it serves as input to another recursive call of the MCTSITERATION algorithm. On return from the recursive call, the returned value V_{child} is accumulated to $Q(s, a)$ (line 14), and the visit count $N(s, a)$ is incremented.

Our Monte Carlo tree search implementation first performs a certain number of MCTS iterations starting from the node corresponding to the current game state in a concrete match. Then it decides on the action that leads to the most frequently visited child node.

Furthermore, our tree search is optimized to reuse the previously built search tree across the moves of a game whenever possible, i.e., when a node corresponding to the current game state is already present in the search tree of the previous move. This optimization avoids performing superfluous MCTS iterations that merely determine previously known information. Instead, it directly builds on already known knowledge, resulting in a more extensive search tree with more information.

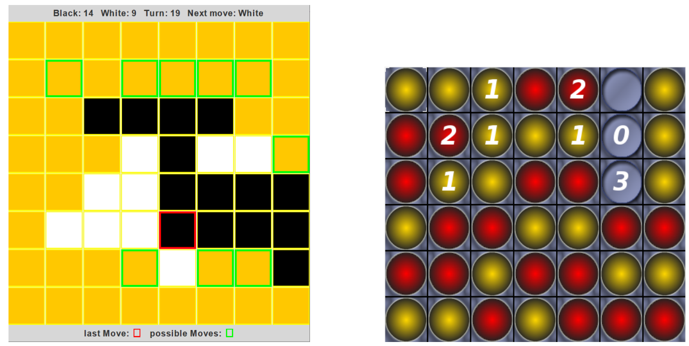
C. N-Tuple Systems

N-tuple systems coupled with TD were first applied to game learning by Lucas in 2008 [16], although n-tuples were already introduced in 1959 for character recognition

game	length n	position m	k	weights	percent active
Othello	7	4	$2 \cdot 100$	3,276,800	51%
ConnectFour	8	4	$2 \cdot 70$	9,175,040	8%
2x2x2 Rubik's	7	{3, 7}	60	3,720,780	31%
3x3x3 Rubik's	7	{2, 3, 8, 12}	120	46,563,392	22%

Table I
N-TUPLE SYSTEMS USED IN THIS WORK. PARAMETERS n, m AND k ARE EXPLAINED IN THE MAIN TEXT. THE NUMBER OF WEIGHTS IS $4^7 \cdot 200$ (OTHELLO) AND $4^8 \cdot 140$ (CONNECTFOUR). FOR 2X2X2 RUBIK'S CUBE, EACH 7-TUPLE HAS EITHER 3 OR 7 POSITIONAL VALUES, DEPENDING ON THE CELL LOCATION. THUS, THE NUMBER OF WEIGHTS DEPENDS ON THE CELL LOCATION. SINCE NOT EVERY WEIGHT REPRESENTS A REACHABLE POSITION, THE NUMBER OF ACTIVE WEIGHTS IS SMALLER, AS GIVEN BY THE PERCENTAGE IN THE LAST COLUMN.

purposes [33]. The remarkable success of n-tuples in learning to play Othello [16] motivated other authors to benefit from this approach for a number of other games. The main goal of n-tuple systems is to map a highly non-linear function in a low dimensional space to a high dimensional space where it is easier to separate 'good' and 'bad' regions. This can be compared to the kernel trick of support-vector machines (SVM). An n-tuple is defined as a sequence of n cells of the board. Each cell can have m positional values representing the possible states of that cell. Therefore, every n-tuple will have a (possibly large) look-up table indexed in form of an n-digit number in base m. Each entry carries a trainable weight. An n-tuple system is a system consisting of k n-tuples. Tab. I shows the n-tuple systems that we use in this work. Each time a new agent is constructed, all n-tuples are formed by *random walk*. That is, all cells are placed randomly with the constraint that each cell must be adjacent to at least one other cell in the n-tuple [21, Appendix C].



(a) (b)

Figure 2. (a) Othello game state. The black cell with a red outline marks the last move of Black. It is White's turn to choose one of the available actions marked by cells with a green border. These actions capture one or more black pieces, which are then flipped to white. (b) ConnectFour game state. It is Red's turn, and he has to place hispiece in the only free column. Subsequently, Yellow wins by reaching *Four in a Row*. Numbers show cell coding: 1 and 2 for players' pieces, 3: empty and reachable, 0: empty, but not reachable (in next move).

D. Temporal Coherence Learning (TCL) and FARL

The TCL algorithm developed by Beal and Smith [28] is an extension of TD learning. It replaces the global learning rate α with the weight-individual product $\alpha\alpha_i$ for every weight w_i. Here, the adjustable learning rate α_i is a free parameter set by a pretty simple procedure: For each weight w_i, two counters N_i and A_i accumulate the sum of weight changes and the sum of absolute weight changes. If all weight changes have the same sign, then $\alpha_i = |N_i|/A_i = 1$, and the learning rate stays at its upper bound. If weight changes have alternating signs, then the global learning rate is probably too large. In this case, $\alpha_i = |N_i|/A_i \to 0$ for $t \to \infty$, and the effective learning rate will be largely reduced for this weight.

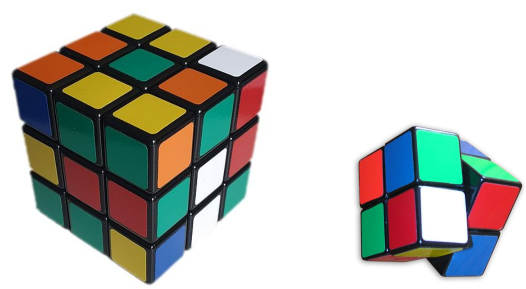
More details on how TCL is coupled to TD n-tuple networks are found in [14]. It was shown in [14] that TCL leads to faster learning and higher win rates for the game ConnectFour.

We also use Final Adaptation RL (FARL) for TD learning, as described in more detail in our previous work [23], [24].

III. EXPERIMENTAL SETUP

A. The Games

1) Othello: (Reversi) is a well-known board game with quite simple rules yet requiring complex strategies to play strong. Fig. 2(a) shows a typical game position. The regular 8x8 Othello has 10^{28} states and an average branching factor of 10. It is an unsolved game (no perfect winning strategy is known).



(a) (b)

Figure 3. (a) Scrambled 3x3x3 Rubik's Cube. (b) 2x2x2 cube (pocket cube) in the middle of a twist.

2) ConnectFour: (Four in a Row) is another board game with quite simple rules. Fig. 2(b) shows a typical end game position. The regular 6x7 ConnectFour has 10^{12} states and a branching factor ≤ 7. It is a solved game: The 1^{st} player wins if playing perfectly.

3) Rubik's Cube: is a famous mathematical puzzle where the goal is to bring an arbitrary scrambled cube (see Fig. 3) by twists into the solved position where each cube face is of unique color. The regular 3x3x3 cube has $4.3 \cdot 10^{19}$ states and a branching factor of 18. The 2x2x2 cube has $3.6 \cdot 10^{6}$ states and a branching factor of 9.

B. Common Settings

We use for all our experiments the same RL agent based on n-tuple systems and TCL. Only its hyperparameters are tuned to the specific game, as shown below. We refer to this agent as ***TCL-base*** whenever it alone is used for game playing. If we wrap this agent by an MCTS wrapper with a given number of iterations, then we refer to this as ***TCL-wrap***.

The hyperparameters for each game were found by manual fine-tuning. This was not too complicated because only a few parameters needed to be changed from their default values. The chosen n-tuple configuration is given in Tab. I, and the remaining parameters are as follows:

- **Othello**: learning rate $\alpha = 0.2$, TCL activated (with default settings), eligibility rate $\lambda = 0.5$, exploration rate $\epsilon = 0.2 \rightarrow 0.1$, 250,000 training episodes.
- **ConnectFour**: learning rate $\alpha = 3.7$, TCL activated (with default settings), eligibility $\lambda = 0.0$, exploration $\epsilon = 0.1 \rightarrow 0.0$, 6,000,000 training episodes.
- **Rubik's Cube**: learning rate $\alpha = 0.25$, TCL activated (with default settings), eligibility $\lambda = 0.0$, exploration $\epsilon = 0.0$, 3,000,000 training episodes.

IV. Results

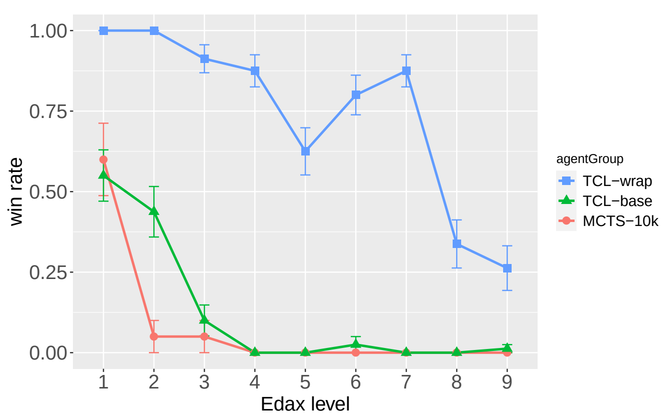


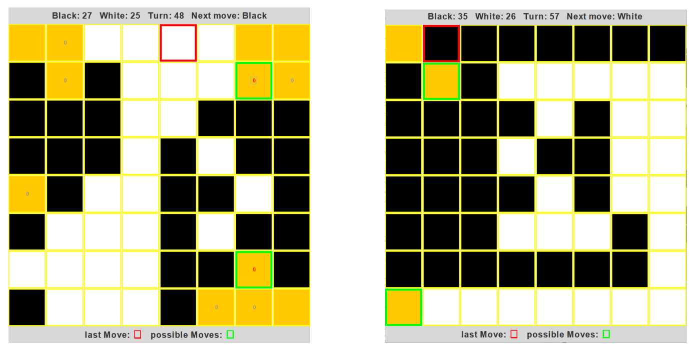
Figure 4. Different Othello agents playing against Edax. *TCL-wrap*: TCL coupled with MCTS wrapper (10,000 iterations); *TCL-base*: TCL alone; *MCTS-10k*: MCTS alone with 10,000 iterations. Error bars show the fluctuations (a) of 20 TCL agents trained with different random n-tuples in the TCL cases and (b) of 20 repeated runs in the MCTS case. Each agent plays in both roles (1^{st} and 2^{nd} player).

A. Othello

It is not too difficult in Othello to reach with game learning algorithms a medium playing strength, i. e. a strength where simple heuristic players are beaten [16], [7]. But it is very difficult to beat the very strong Othello playing program Edax [11]. Edax has a configurable playing strength (level, depth) between 0 and 60. Only a few Othello agents can beat Edax beyond level 2.

We compare our agents with Edax at different levels. Since all agents (Edax, *TCL-base* and *TCL-wrap*) are deterministic move predictors, repeated evaluation runs with the same pair of agents always yield the same results and cannot be used to collect statistics. We use the following procedure to get statistically sound results: We draw 20 different random n-tuple configurations (*random walk*, see Sec. II-C) and train for each configuration a separate *TCL-base* agent. As a byproduct, this will also show that random n-tuple configuration does not lead to too large variations.

Fig. 4 shows the results obtained: Both MCTS and *TCL-base* cannot defeat Edax at level 2 and beyond (their win rates are lower than 50% from level 2 on). The situation changes dramatically as soon as we wrap *TCL-base* by MCTS: *TCL-wrap* defeats Edax up to level 7 and has a win rate above 25% for levels 8 and 9.



(a) (b)

Figure 5. Tactics of Edax in Othello: (a) Move 48 in a game *TCL-base* (Black) vs. Edax level 7 (White): It is Black's turn, and Edax forces Black into disadvantageous moves that allow White to capture the corners. *TCL-wrap* will avoid such disadvantageous positions. (b) Move 57 in a game *TCL-wrap* (Black) vs. Edax level 8 (White): Now it is White's turn, and although Black has the current majority of pieces, White will eventually win because Black has to pass and White moves *three times in a row*.

Interpretation: What are the reasons for opponents to win or lose in Othello against Edax? – To investigate this, we analyze specific Othello episodes: When Edax plays at level 7, it has advanced tactics that narrow the range of possible actions for the opponent (Fig. 5(a)): If Edax (2^{nd}) plays against opponent *TCL-base* (1^{st}), Edax forces *TCL-base* towards the end of the episode to play disadvantageous moves. If we now replace the opponent (1^{st}) with *TCL-wrap*, it avoids these traps: The planning stage of *TCL-wrap* helps to foresee the disadvantageous positions when they are some moves ahead; now *TCL-wrap* finds other moves to avoid them and is thus not forced into the disadvantageous positions.

At level 8 or higher, Edax shows another tactic: It plays in such a way that the last 2-4 moves are pass moves for the opponent (Fig. 5(b)): Since the opponent has no available action at its disposal, it is forced to pass the move right to Edax again. During the very last moves of an episode, Edax will thus gain the majority of pieces. Currently, *TCL-wrap* is not able to avoid these pass situations, at least not in the majority of the episodes played.

W/T/L		2nd player				won games rate
		TCL-wrap	AB-DL	*TCL-base*	MCTS	
1^{st}	*TCL-wrap*		99/0/**1**	100/0/0	100/0/0	66.3%
	AB-DL	100/0/0		100/0/0	100/0/0	64.9%
	TCL-base	100/0/0	91/2/**7**		100/0/0	49.0%
	MCTS	**1**/2/97	17/3/80	**97**/2/1		19.8%

Table II
RESULTS OF A CONNECTFOUR TOURNAMENT WITH 4 AGENTS. SHOWN IS THE W/T/L (WIN/TIE/LOSS) COUNT OF THE 1^{st} (ROW) PLAYER WHEN PLAYING 100 EPISODES AGAINST THE 2^{nd} (COLUMN) PLAYER. AGENTS ARE RANKED BY THEIR OVERALL RATE OF GAMES WON (LAST COLUMN). THE COLORED AND BOLD NUMBERS SHOW REMARKABLE IMPROVEMENTS OF *TCL-wrap* OVER *TCL-base*.

B. ConnectFour

ConnectFour is a non-trivial game that is not easy to master for humans. However, its medium-size complexity allows for very strong tree-based solutions when combined with a pre-computed opening book. These near-perfect agents are termed AB and AB-DL since they are based on alpha-beta search (AB) that extends the Minimax algorithm by efficiently pruning the search tree. Thill et al. [15] were able to implement alpha-beta search for ConnectFour in such a way that it plays near-perfect: It wins all games as 1^{st} player and wins very often as 2^{nd} player when the 1^{st} player makes a wrong move. AB and AB-DL differ in the way they react to losing states: While AB just takes a random move, AB-DL searches for the move, which postpones the loss as far (as distant) as possible (DL = distant losses). It is tougher to win against AB-DL since it will request more correct moves from the opponent and will very often punish wrong moves.

We perform a tournament with the following 4 agents:

- ***TCL-wrap***: MCTSWrapper[*TCL-base*] (iter=1,000, c_{PUCT}=1.0, unlimited depth),
- ***TCL-base***: TCL alone,
- **AB-DL**: Alpha Beta with Distant Losses,
- **MCTS**: MCTS(UCT, random playouts, iter=10,000, treeDepth=40)

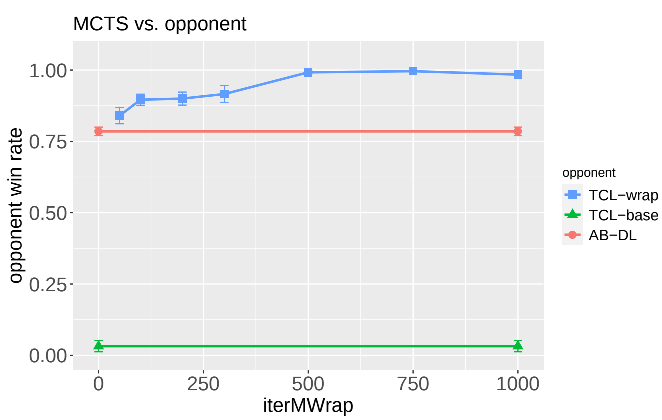


Figure 6. The effect of MCTS wrapping on ConnectFour. Shown are the averages from 5 runs, where each run consists of 25 competition episodes MCTS (1^{st}) vs. opponent (2^{nd}). The opponents are a) *TCL-wrap*: TCL wrapped by MCTS wrapper with iterMWrap iterations; b) *AB-DL*: Alpha-Beta agent with distant losses; c) *TCL-base*: TCL without MCTS wrapper.

The results are shown in Tab. II and can be described as follows: *TCL-wrap* and AB-DL win nearly all their games when playing first (ConnectFour is a theoretical win for the 1^{st} player). *TCL-base* (1^{st}) wins against AB-DL (2^{nd}) the majority of its games (91%), but not all. If we enhance *TCL-base* by MCTS wrapper, the win rate of *TCL-wrap* rises to fantastic 99%, so it avoids 8/9 of the former *TCL-base* losses.

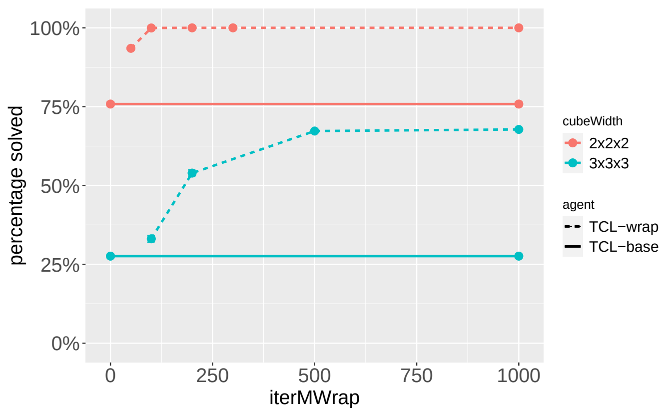
MCTS as the weakest agent in the tournament, wins as 1^{st} player most of its games (97%) against *TCL-base* (2^{nd}), but it predominantly loses against *TCL-wrap* and AB-DL (2^{nd}). *TCL-wrap* as 2^{nd} player is in this respect significantly stronger than AB-DL (97% vs. 80% win rate, resp.), which leads for *TCL-wrap* to a higher total rate of 66.3% won games as compared to AB-DL (64.9%). Besides that, the total won games rate 66.3% is a big jump forward when compared to the total won game rate 49% of *TCL-base*.

Interpretation: MCTS plays differently, perhaps more surprising, than near-optimal agents. Since *TCL-base* was trained on a near-optimal agent (itself), it has never seen the 'surprising' moves of MCTS and will probably often react wrongly on that moves. Thus, *TCL-base* loses most of its games when playing 2^{nd}. If we now add with MCTS wrapper a planning component to *TCL-base*, then *TCL-wrap* can find better responses to the 'surprising' moves, and it can better exploit the occasional wrong moves of MCTS. As a consequence, it wins most of the episodes.

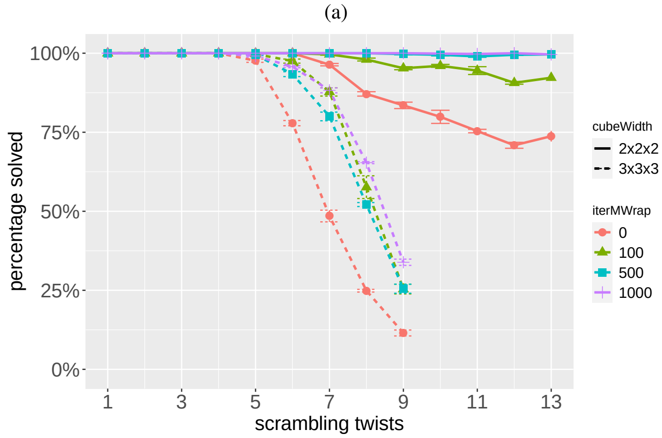
Fig. 6 shows the results of MCTS-wrapping in ConnectFour as a function of MCTS wrapper iterations. Even a small amount of iterations (50-100) already leads to a *TCL-wrap* win rate of $> 80\%$. With 500 iterations or more, *TCL-wrap* achieves a win rate near 100%.

C. Rubik's Cube

We investigate two variants of Rubik's Cube: 2x2x2 and 3x3x3. We trained TCL agents by presenting them cubes scrambled with up to p_{max} twists where $p_{max} = 13$ for 2x2x2 and $p_{max} = 9$ for 3x3x3. This covers the complete cube space for 2x2x2, but only a small subset for 3x3x3, where God's number [34] is known to be 20. We evaluate the trained agents on 200 scrambled cubes that are created by applying a given number p of scrambling twists to a solved cube. The agent now tries to solve each scrambled cube. A cube is said to be *unsolved* if the agent cannot reach the solved cube in $e_E = 20$ steps. More details on our method are found in [35].



(a)



(b)

Figure 7. The effect of MCTS wrapping on Rubik's Cube. Shown are the averages from 4 runs, where each run evaluates the ability of the agents to solve a large set of scrambled cubes (a) as a function of MCTS iterations: 2x2x2: 600 cubes scrambled with either 11, 12 or 13 twists; 3x3x3: 600 cubes scrambled with either 7, 8 or 9 twists; (b) as a function of scrambling twist: 200 scrambled cubes for each twist number.

Here we are interested in the relative strength of agents with and without MCTS-wrapping. The results are shown in Fig. 7: While *TCL-base* could only solve 75% (2x2x2) or 25% (3x3x3) of the scrambled cubes, resp., the MCTS-wrapped agent *TCL-wrap* could either fully solve the problem (2x2x2) or at least double or triple the percentage of solved cubes (3x3x3).

Interpretation 2x2x2: Since the solved-rate of *TCL-base* is only 75%, the value function $V(s)$ does not predict for every state s the right action (resulting in a short path to the solved cube). However, if we add the planning stage of MCTS-wrapper, then the action with the highest $V(s)$ after a few 'what-if' steps is selected. This is sufficient to boost the solved-rate to 100% after 200 or more MCTS-iterations.

Interpretation 3x3x3: The agent has seen during training only a small subset of cubes with up to 9 scrambling twists. Therefore, the solved-rates for $p = 8, 9$ are much lower in the *TCL-base* case because it is very likely that the cube 'escapes' with a wrong move into the unknown area of $p = 10$ or higher. It is interesting to see that the MCTS planning stage can double or triple the solved-rate. However, it can not cure everything since the high branching factor of 18 together with slight inaccuracies of the value function approximator makes it likely that even 1000 iterations of MCTS-wrapper do not explore enough to find the right path.

game	n_{ag}	base training time	i_{MCTS}	*factor*	hypothetical wrapped training time
Othello	20	1.5 d	10,000	2,575	10.6 years
ConnectFour	10	1.4 d	1,000	850	3.3 years
RubiksCube	5	2.2 h	1,000	770	71 days

Table III
TRAINING TIMES TO TRAIN n_{ag} AGENTS WITHOUT AND WITH MCTS WRAPPING (HYPOTHETICAL) FOR ALL GAMES.

D. Computation times

The MCTS wrapper for RL agents, as proposed in this paper, has the advantage that it does not cost any additional training time since it is an enhancement added *after* agent training.

The extra computational resources needed during game play or evaluation are moderate. This is because there are usually only a few evaluation episodes (compared to the huge number of training episodes) and because the MCTS wrapper does not require too many iterations.

The above advantage becomes more apparent if we compare the actual *TCL-base* training times with the would-be training times if the MCTS planning stage were also used during training, as shown in Tab. III: The base training time is the time actually needed to train n_{ag} agents without MCTS wrapper. All computations were done on a single CPU Intel i7-9850H @ 2.60GHz.[2]

The wrapped training times are estimated by multiplying the base training time with *factor* which is established by running a few episodes without and with MCTS wrapper doing i_{MCTS} iterations. This estimate rests on the assumption that a wrapped agent needs as many training episodes as a base agent. This assumption is reasonable because the exploration of the state space normally dictates the number of episodes needed. But it was not proven empirically because the hypothetical training times are astronomic: We see from Tab. III that with the same hardware, many years or at least near 100 days of computation time would be necessary. Of course, large speed-ups would be possible if dedicated hardware or parallel execution on many cores were used, but often this hardware is just not available.

V. Discussion

A. Comparison with Other N-Tuple Research

There are two papers in the game learning literature that connect n-tuple networks with MCTS: Sironi et al. [36] use the n-tuple bandit EA to automatically tune a self-adaptive MCTS. This is an interesting approach but for a completely different goal and not related to AlphaZero. Chu et al. [17]

[2]To get a single-agent training time, the base training time has to be divided by n_{ag} which results for example in 1.8 hours training time for one Othello agent.

use an n-tuple network as a heuristic selector for MCTS in the game EWN. Although they pursue a similar goal as our work (*'predict with MCTS + n-tuple a good next move'*), they follow a different path since they do not incorporate reinforcement learning and do not follow the AlphaZero approach.

So – to the best of our knowledge – this work is the first to couple n-tuple networks with MCTS using the AlphaZero approach.

B. Comparison with Other RL Research

In this section, we compare our results with other RL approaches from the literature.

Dawson [37] introduces a CNN-based and AlphaZero-inspired [2] RL agent named ConnectZero for the game ConnectFour, against which can be played online. Although it reaches a good playing strength against MCTS_{1000}, it is inferior to our AlphaBeta-DL and *TCL-wrap*: We performed 10 episodes with ConnectZero starting (which is a theoretical win), but found instead that AlphaBeta playing second won 80% of the episodes and *TCL-wrap* playing second won all episodes. This is in contrast to our *TCL-base* and *TCL-wrap*, which win nearly all episodes when starting against AlphaBeta-DL (see Tab. II).

Concerning the game Othello, there are a number of other researchers that do RL-based game learning: van der Ree and Wiering [26] reached in 2013 with their Q-learning agent against the heuristic player BENCH (positional player) a win rate of 87%. We reach with both *TCL-base* and *TCL-wrap* a win rate of 100% against BENCH. Liskowski et al. [12] show in Table IX that their agent wins against Edax up to and including Edax level 2. We win up to and including Edax level 7.

In 2021, Norelli and Panconesi [10] obtained with their system OLIVAW the best Othello results up-to-date: It defeats Edax up to and including Edax level 10. This is a truly impressive result, but it also took considerable computational resources to achieve it: Although much cheaper than DeepMind's original AlphaZero, they needed an informal crowd computing project with 19 people for game generation and then about 30 days to train a single agent on Google Colaboratory using GPU and TPU hardware. Thus, fine-tuning of hyperparameters or ablation studies could not be undertaken.

In our work presented here, we defeat Edax only up to level 7, but with a much simpler architecture that is trainable in less than 2 hours on a single standard CPU. It is, on the one hand, interesting that our architecture, which keeps the costly MCTS completely out of the training process, can get so far.

On the other hand, there is, of course, a performance gap to [10], and it would be interesting to investigate which element of the more complex architecture in [10] is responsible for the performance gain. We see here two hypothetical candidates: First, including MCTS in the training phase leads to better positional material in the replay buffer. Second, the network architecture of OLIVAW uses a Residual Network, a somewhat reduced version of the original AlphaZero Residual Network, but still a deeper architecture than our n-tuple network.

Concerning the puzzle Rubik's Cube, the pioneering work of McAleer [38] and Agostinelli [39] in 2018 and 2019 shows that the 3x3x3 cube can be solved without putting human knowledge or positional-pattern databases into the agent. They solve arbitrary scrambled cubes with a method that is partly inspired by AlphaZero but also contains special tricks for Rubik's Cube. The deep network used in [38] had over 12 million weights and was trained for 44 hours on a 32-core server with 3 GPUs. Our approach can solve the 2x2x2 cube completely, but the 3x3x3 cube only partly.

VI. CONCLUSION AND FUTURE WORK

We have shown on the three challenging games, Othello, ConnectFour, and Rubik's Cube, that an AlphaZero-inspired MCTS planning stage boosts the performance of TD-n-tuple networks. Interestingly, this performance boost is even reached when MCTS is *not* part of the training stage, which leads to very large reductions in training times and computational resources.

The new architecture was evaluated on the three games without any game-specific changes. We reach perfect play for ConnectFour and 2x2x2 Rubik's Cube. For the games Othello and 3x3x3 Rubik's Cube, we observe good results and increased performance compared to our version without MCTS planning stage, but we do not reach the high-quality results of Norelli and Panconesi [10] on Othello (beats Edax level 10 where we reach only level 7) and of Agostinelli, McAleer et al. [39], [38] on 3x3x3 Rubik's Cube (they solve all scrambled cubes while we solve only cubes with up to 9 twists). Both high-performing approaches require considerably more computational resources.

It is an interesting topic of future research to investigate which element of the more complex architecture (MCTS in the training phase or deep residual network for the approximator) is more relevant to reach these impressive high-quality results. However, our smaller-sized architecture has the advantage to allow faster training and more parameter tuning on mid-complex games with simpler hardware accessible to everyone.

We also plan to extend our MCTS wrapper concept to non-deterministic games (e.g., EWN, 2048, Blackjack, Poker) where previous research [40] has shown that plain MCTS is not sufficient and has to be extended by the Expectimax approach.

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