

Analysing Math and Strategy in Games

- Work in progress on the French Military Game

Lisa Rougetet : lisa.rougetet@univ-brest.fr

Eric Piette : eric.piette@maastrichtuniversity.nl

DLP Workshop – 12th April 2022



Centre François Viète



Université de Bretagne Occidentale



**Maastricht
University**



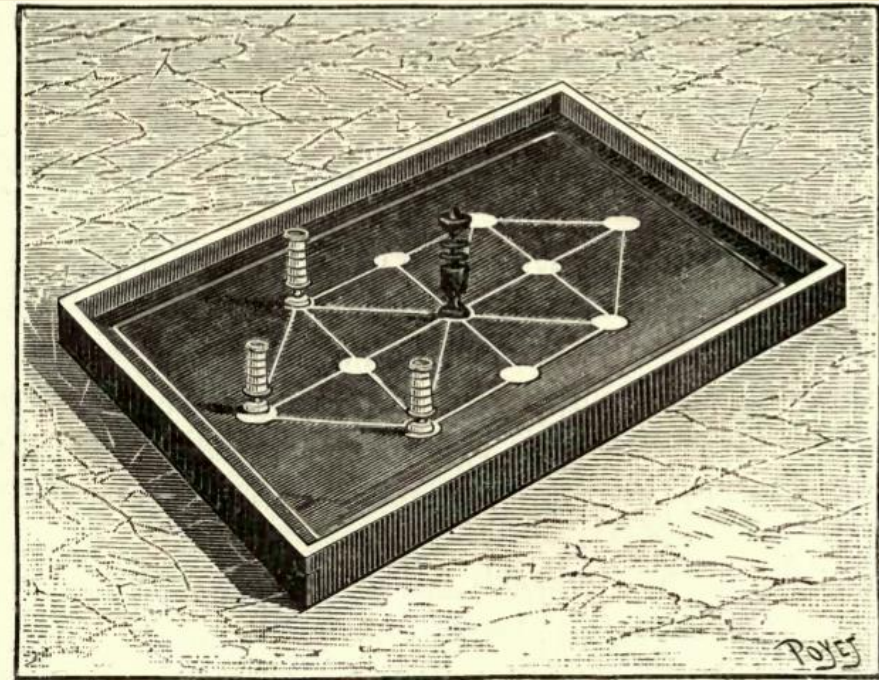
**Digital
Ludeme
Project**



European Research Council
Established by the European Commission

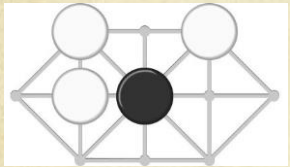
The French Military Game

- Asymmetrical blockade game

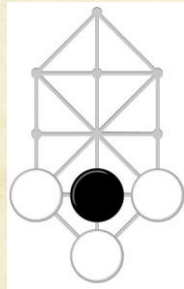


The French Military Game

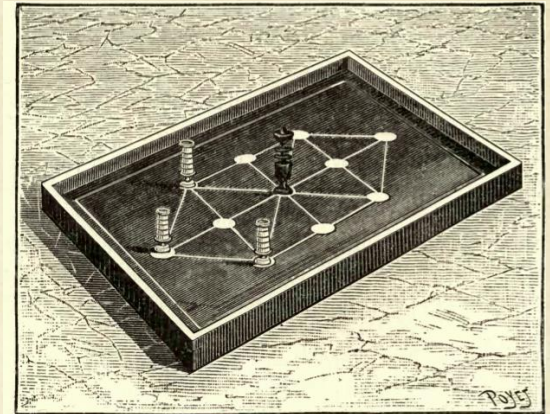
- Asymmetrical blockade game.
- Many other games share the same board:



Haretaavl
Modern
Northern Europe,
Western Europe
Board, Hunt

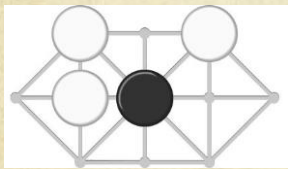
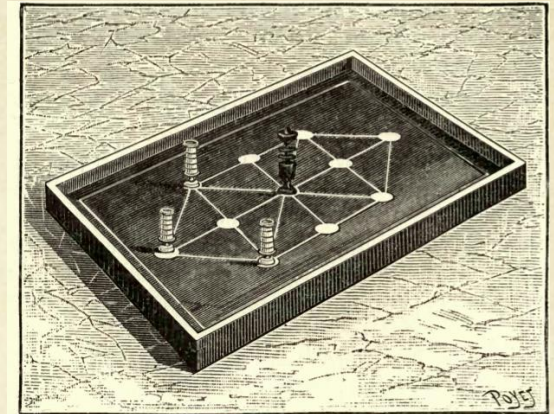


La Liebre Perseguida
Modern
Southern Europe,
Western Europe
Board, Hunt

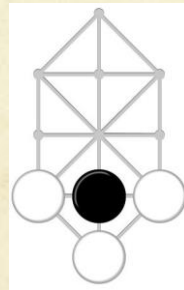


The French Military Game

- Asymmetrical blockade game.
- Many other games share the same board:

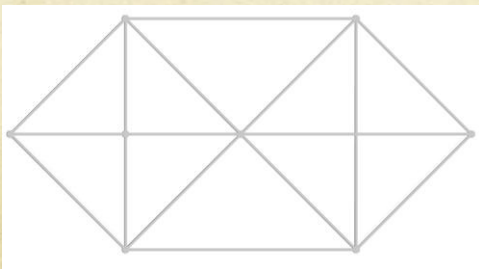


Haretafl
Modern
Northern Europe,
Western Europe
Board, Hunt

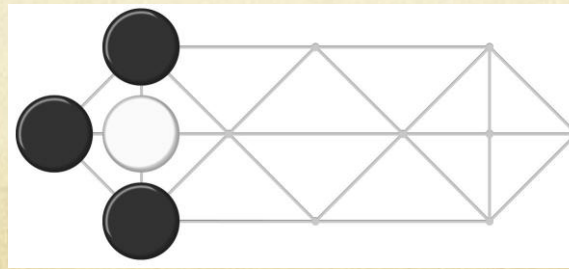


La Liebre Perseguida
Modern
Southern Europe,
Western Europe
Board, Hunt

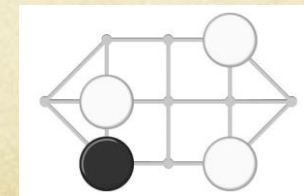
- Or quite “similar” boards:



The board with 9 points



A board with 12 points from Vendsyssel, Northern Jutland

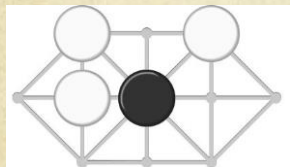


Jänes Soppi
Modern
Northern Europe
Board, Hunt

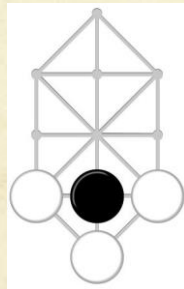
(Michaelsen, 2014)

The French Military Game

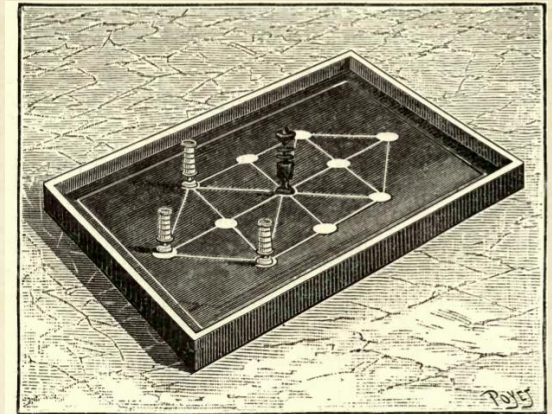
- Asymmetrical blockade game.
- Many other games share the same board:



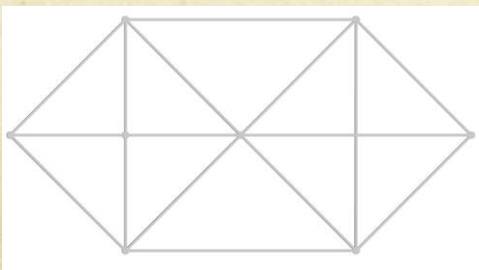
Haretavl
Modern
Northern Europe,
Western Europe
Board, Hunt



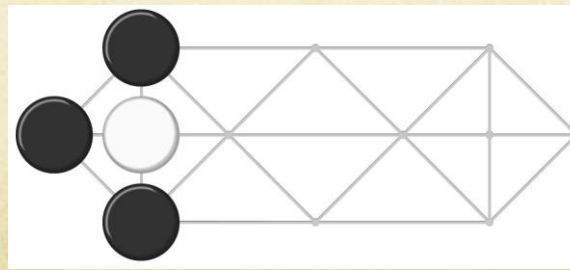
La Liebre Perseguida
Modern
Southern Europe,
Western Europe
Board, Hunt



- Or quite “similar” boards:



The board with 9 points



A board with 12 points from Vendsyssel, Northern Jutland

→ See Ludii
portal 😊

(Michaelsen, 2014)

Historical insight of the FMG

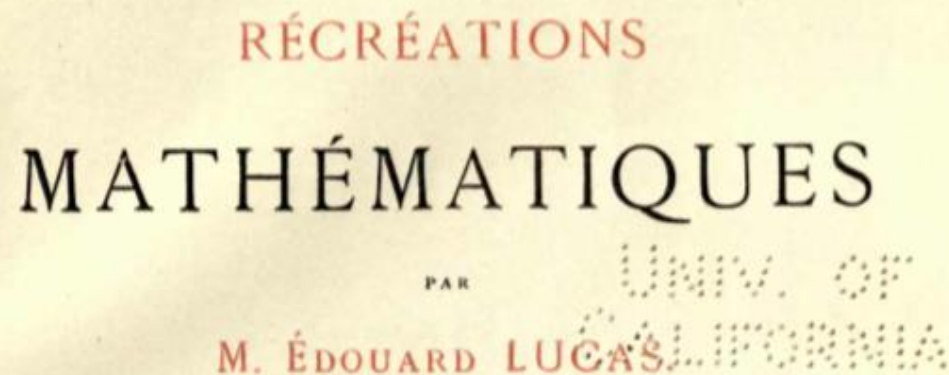
- Described (and analyzed) for the 1st time by the French mathematician Édouard Lucas.



Édouard Lucas (1842-1891)

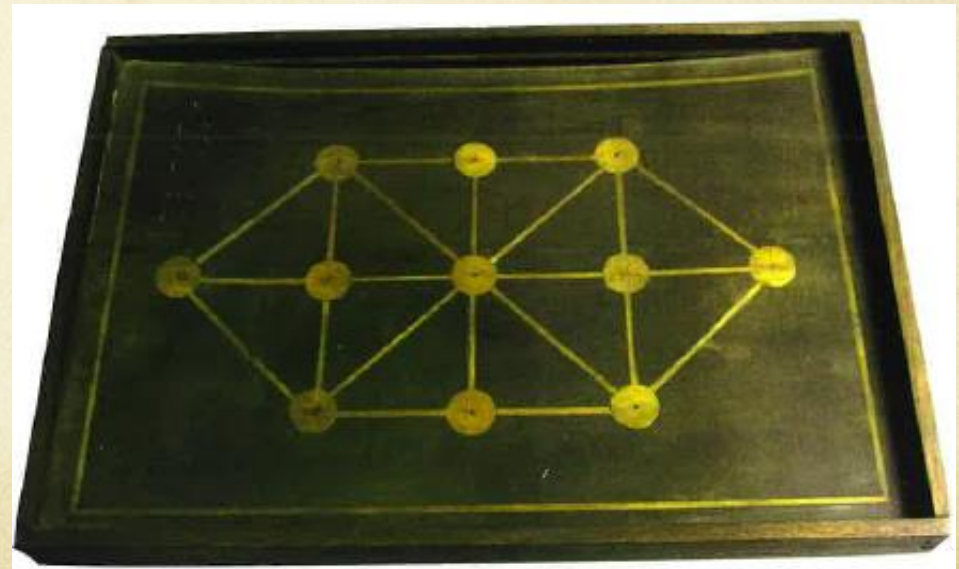
Historical insight of the FMG

- Described (and analyzed) for the 1st time by the French mathematician Édouard Lucas.
- *La Nature*, 15^e année, N°756, 26 novembre 1887, pp. 402-404.
- *Récréations mathématiques Volume 3*, 1893, Paris : Gauthier-Villars, pp. 107-116.



Historical insight of the FMG

« Less difficult than chess, the “Jeu militaire” is among the most instructive ones, and deserves to be recommended as a very useful distraction for officers and non-commissioned officer. » (Lucas, 1893)

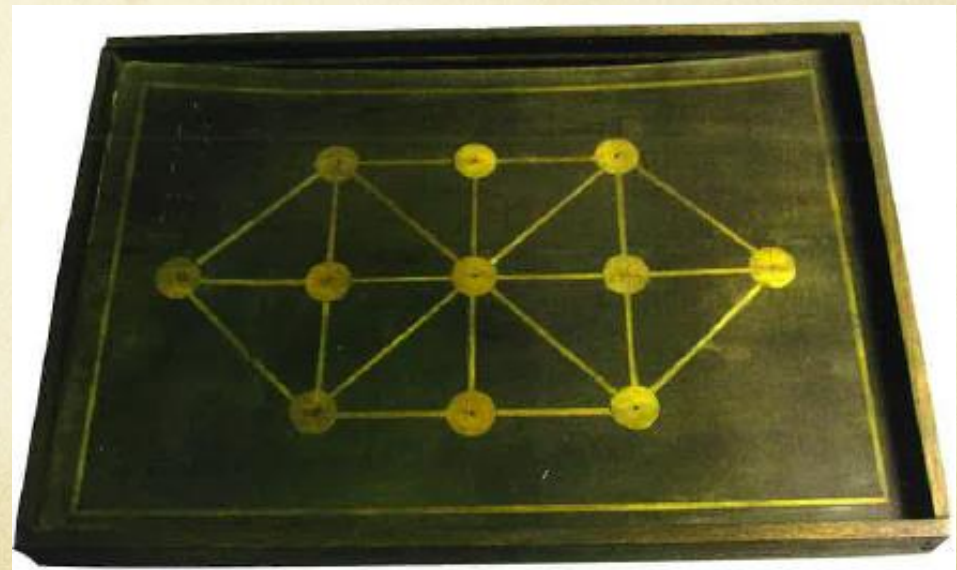
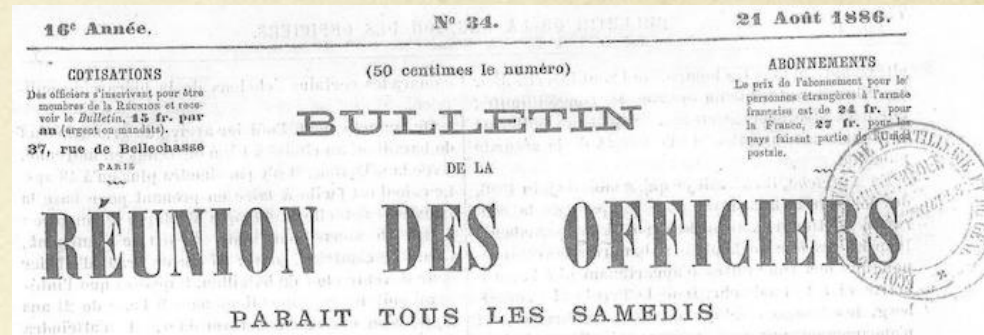


Board of the FMG (1887) at the CNAM, Paris. © M. Boutin

Historical insight of the FMG

« Less difficult than chess, the “Jeu militaire” is among the most instructive ones, and deserves to be recommended as a very useful distraction for officers and non-commissioned officer. » (Lucas, 1893)

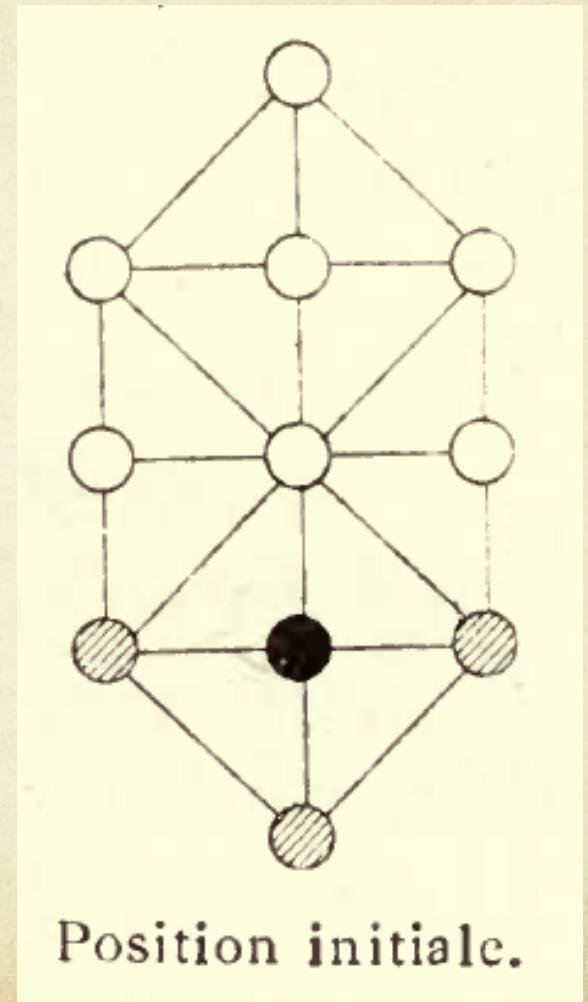
- Mentioned in a military newspaper in 1886.



Board of the FMG (1887) at the CNAM, Paris. © M. Boutin

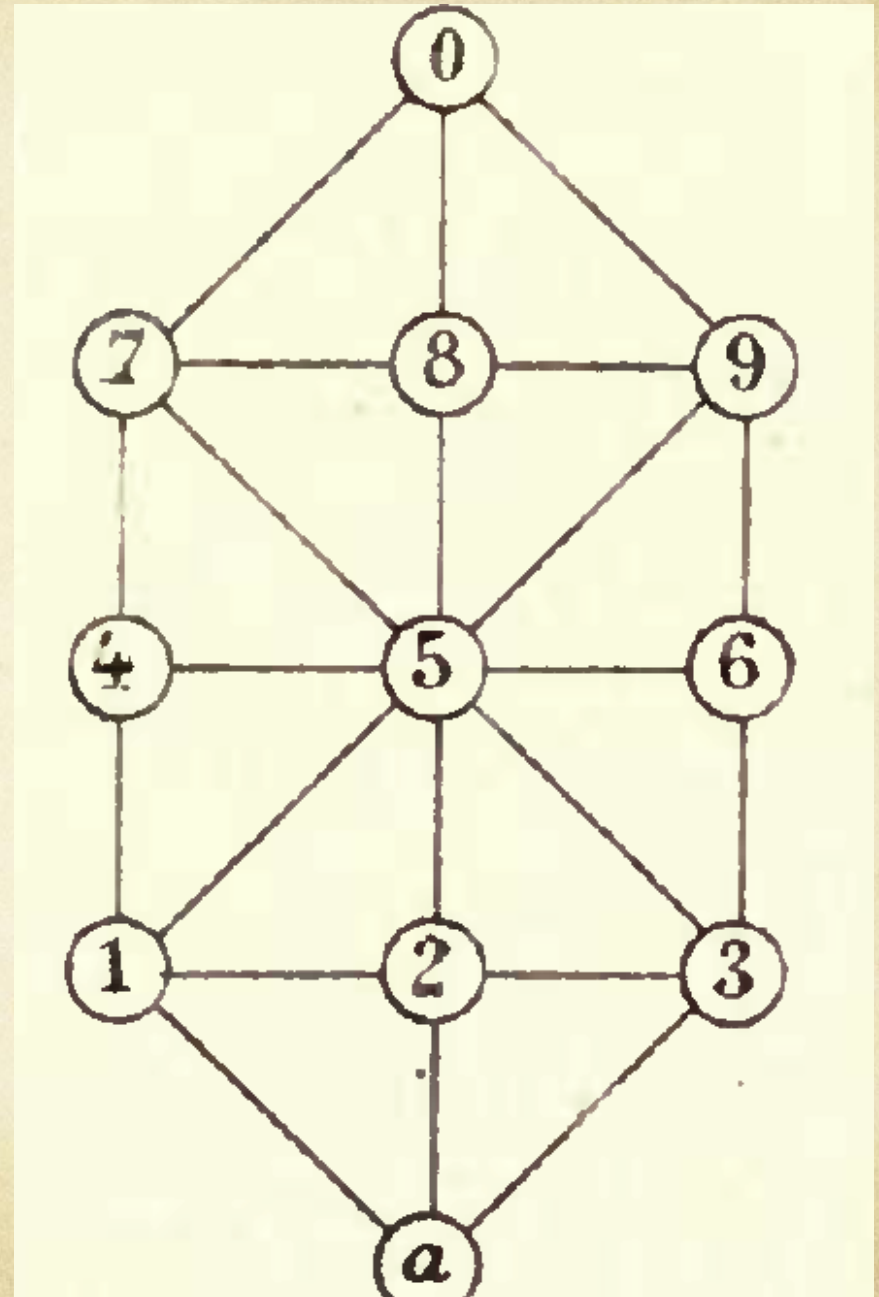
The French Military Game: rules

- The army = black token (P1)
- The towers = white tokens (P2)
- **Aim of the army:** not being blocked by the towers.
- **Aim of the towers:** block the army.
- Moves :
 - The army (P1) can move one step in any direction.
 - Towers can move one step only forward, diagonally and laterally (one tower at a time).



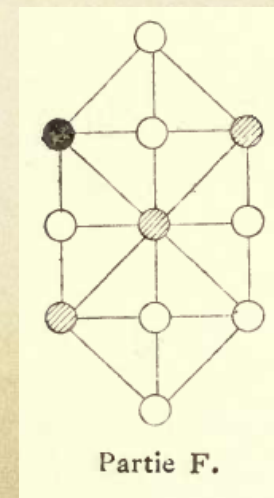
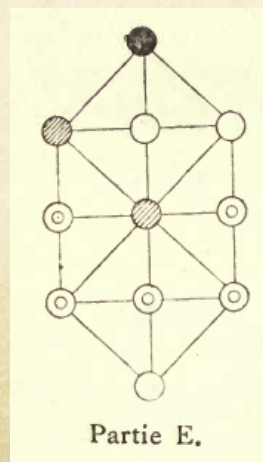
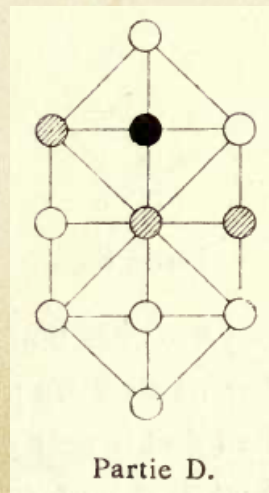
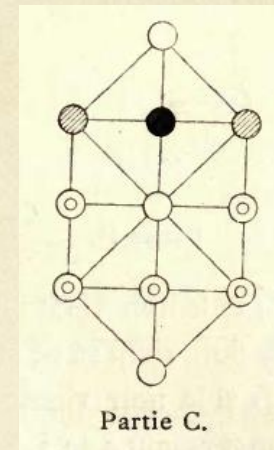
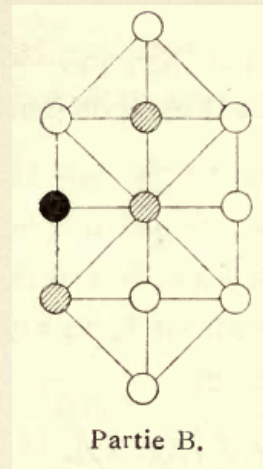
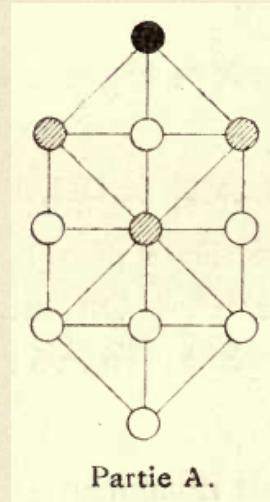
Mathematical analysis of Lucas

- Positions are numbered.
- Initial position: black on 2 and whites on 1, a and 3.
- Notation : $1a3 - 2$



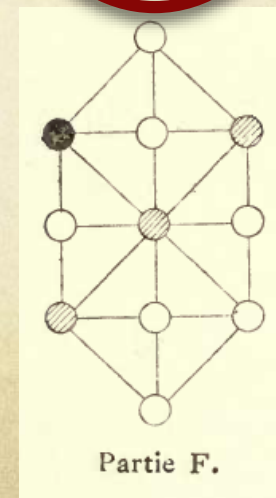
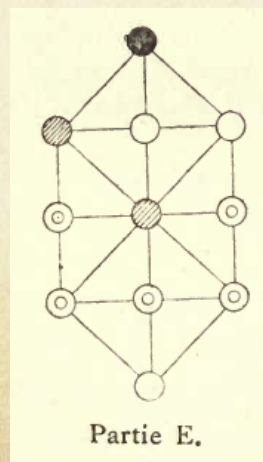
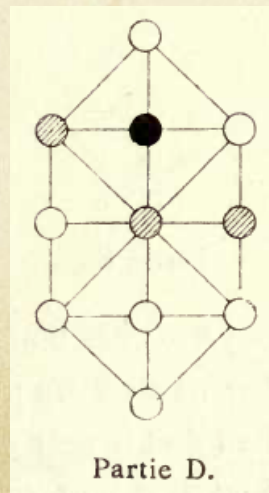
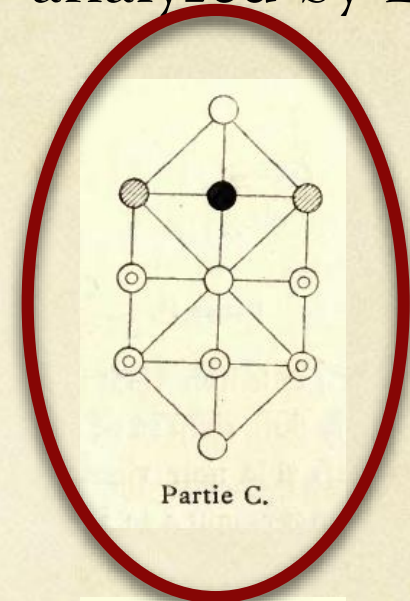
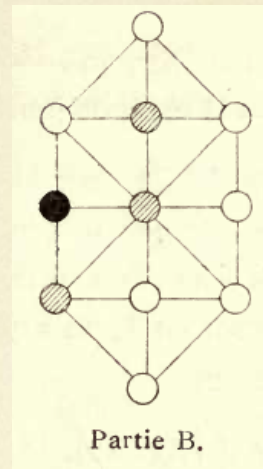
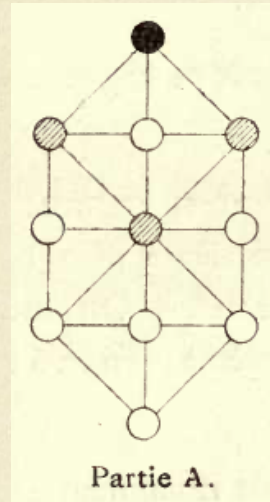
Mathematical analysis of Lucas

- 6 endgames are identified and completely analyzed by Lucas.



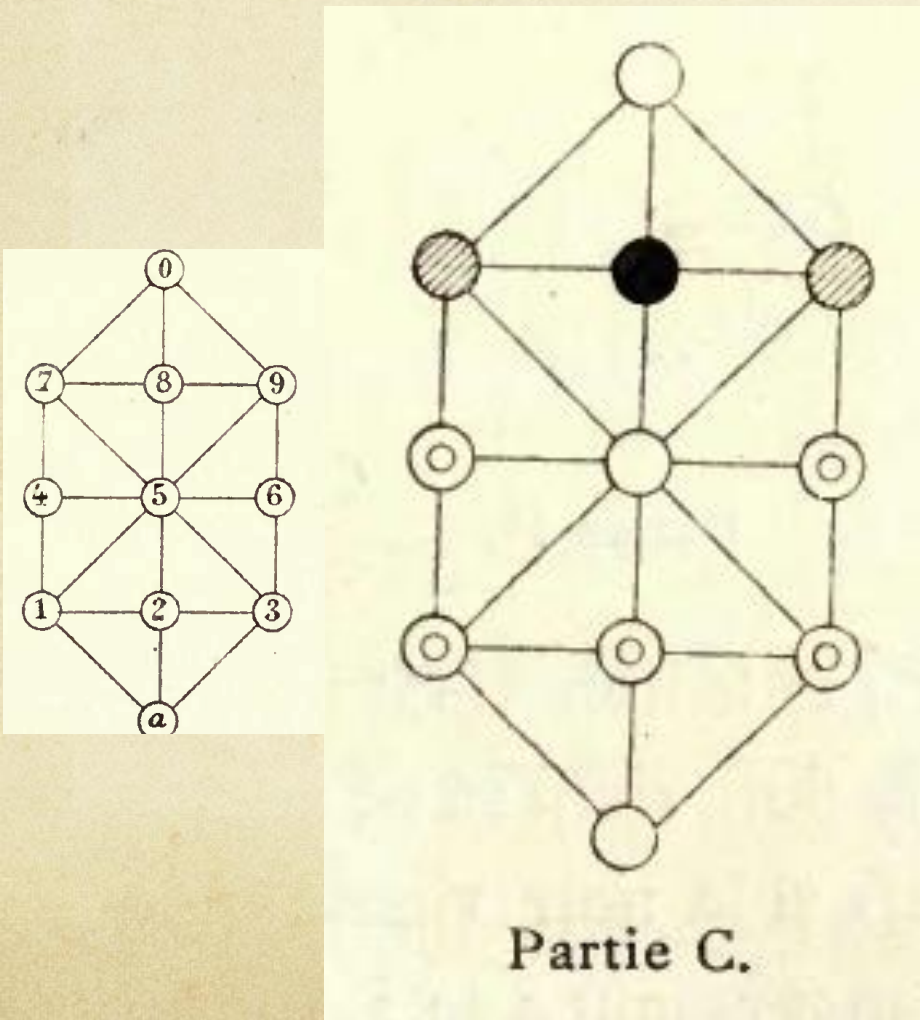
Mathematical analysis of Lucas

- 6 endgames are identified and completely analyzed by Lucas.



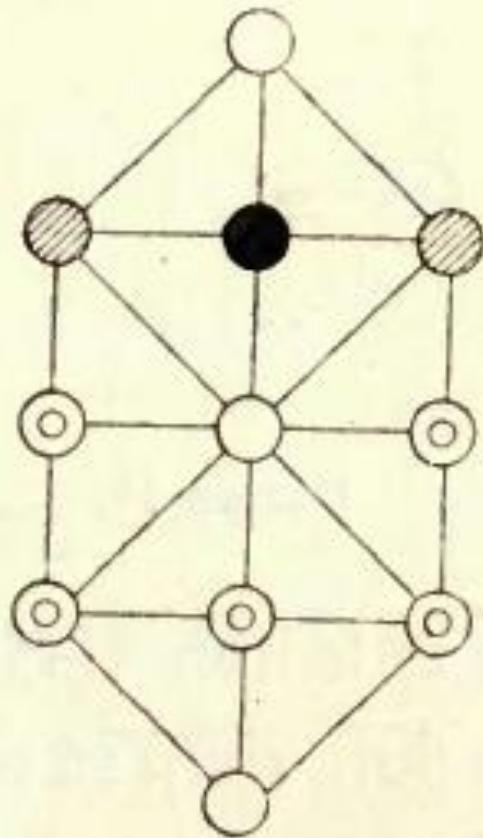
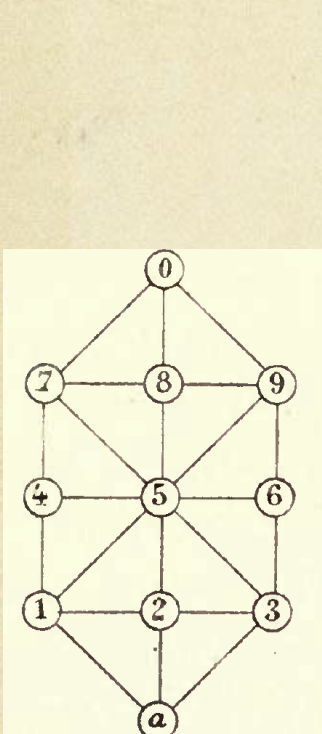
Mathematical analysis of Lucas

- 6 endgames are identified and completely analyzed by Lucas.



Mathematical analysis of Lucas

- 6 endgames are identified and completely analyzed by Lucas.

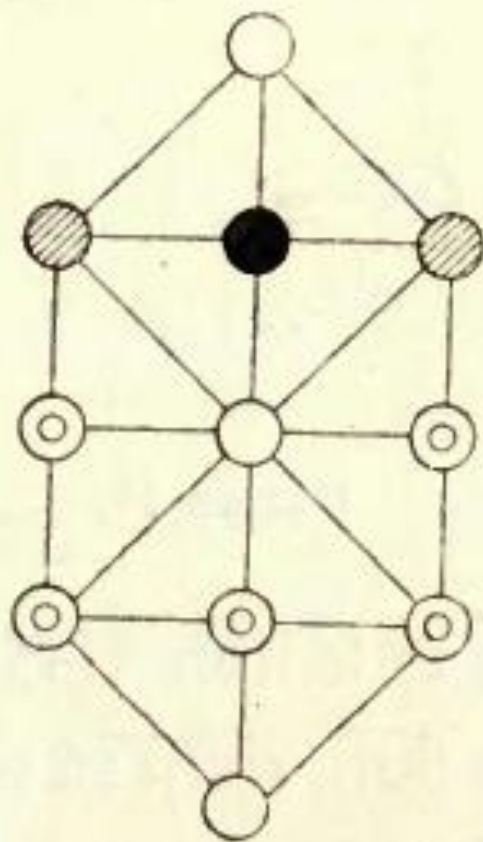
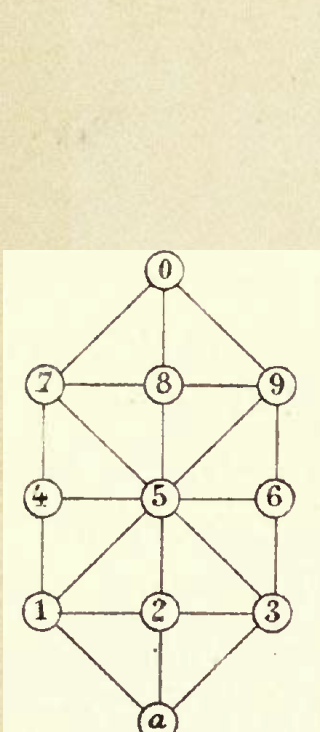


Partie C.

→ White wins in two moves.
(The 3rd tower can be anywhere on the spotted positions.)

Mathematical analysis of Lucas

➤ 6 endgames are identified and completely analyzed by Lucas.



Partie C.

➔ White wins in two moves.

(The 3rd tower can be anywhere on the spotted positions.)

➤ White plays the 3rd tower on the center (5).

➤ Black plays on 0.

➤ White moves the tower from 5 to 8.

Notation: 179 - 8

Mathematical analysis of Lucas

- 6 endgames are identified and completely analyzed.
- Lucas provides an array which summarizes all the possible games.

123	4 135	7 345	8 357	0 E			
				9 F			
			0 349	7 459	8 D		
					0 E		
				8 359	7 259	4 159	7 F
						8 159	7 F
							0 E
						0 E	
	7 125	4 135 ※			0 E		
		8 135	7 345 ※				
			0 138	7 158	4 B		
					0 258	7 259 ※	
		0 129	7 259 ※				
			8 159 ※				
	8 135 ※						

123		4 135	7 345		8 357		0 E	
							9 F	
					0 349	7 459	8 D	
							0 E	
					8 359	7 259	4 159	7 F
							8 159	7 F
								0 E
							0 E	
		7 125	4 135 ✕				0 E	
			8 135	7 345 ✕				
				0 138	7 158	4 B		
						0 258	7 259 ✕	
			0 129	7 259 ✕				
				8 159 ✕				
		8 135 ✕						

Towers can win in
12 moves max.

Mathematical analysis of Lucas

- 6 endgames are identified and completely analyzed.
- Lucas provides an array which summarizes all the possible games.

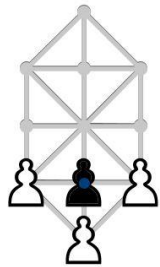
123	4 135	7 345	8 357	0 E					
				9 F					
			0 349	7 459	8 D				
					0 E				
				8 359	7 259	4 159	7 F		
						8 159	7 F		
							0 E		
						0 E			
	7 125	4 135 *			0 E				
		8 135	7 345 *						
			0 138	7 158	4 B				
					0 258	7 259 *			
		0 129	7 259 *						
			8 159 *						
	8 135 *								

« In few hours we can master the game, and win the prize of hundred “francs” offered by the leaflet as many times as we want. But it’s about finding the address of the banker; this problem is more difficult than the previous one. » (Lucas, 1893)

Ludii's verification by Eric ☺

Jeu Militaire (FMG) in Ludii

Jeu Militaire (French Military Game, Hare and Hounds) DLP Game



1 Player 1
2 Player 2 (Ludii) ☺



Category [Board](#), [Hunt](#)

Description

The Jeu Militaire is a game documented since the nineteenth century in France. It was reported by several sources as being played in the French military.

Rules

The board consists of three intersecting lines, with diagonals drawn in the square formed. On opposite ends of the square, the central line is extended beyond the square and lines drawn from the adjacent corners to the end of this line, forming two triangles at opposite ends. One player takes the part of three white towers, the other the part of a single black army piece. First the army moves their piece, then the towers move one of their pieces, play alternating thereafter until the game is ended. A tower piece may move one step along a marked line in any forward or sideways direction. Tower pieces cannot move backwards, diagonally or otherwise, towards the end of the board from which they started. The army may move one step in any direction along a marked line. The army wins by passing the towers and reaching the end of the board from which they started. The towers win by trapping the army so that it cannot move in its turn.

Lucas 1887: 402-404.

These rules were taken from the [Lucas](#) ruleset.

All Rulesets

[Described rulesets](#)

[Lucas](#) Rules for the French Military Game.

[Suggested rulesets](#)

[Gardner](#) Described by M. Gardner.

Origin

France

Ludeme Description

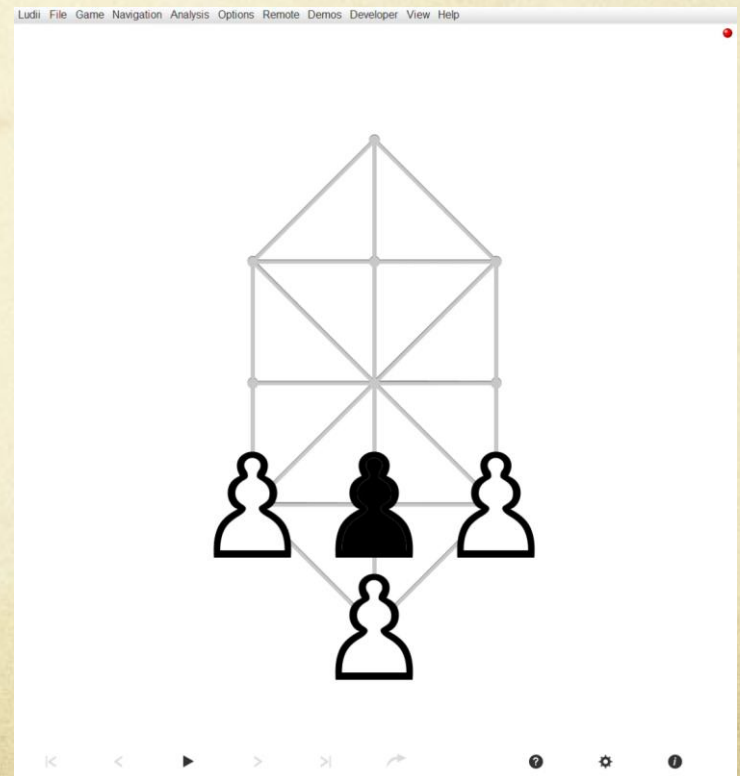
[Jeu Militaire.lud](#)

Concepts

[Browse all concepts for Jeu Militaire here.](#)

Estimation	30s, ~ 800,000 p
Game Length	~18.32
Branching Factor	~4.22
Game-tree Complexity	10^{11}

Forced Win for White (as described by Lucas)



Concepts analysis of FMG (Few extracts)

Numerical concepts	Value
Num Players	2
Num Playable Sites	11
Num Columns	3
Num Rows	5
Avg Directions	4
Num Start Components	4
Num Component Types	2
Num Top Sites	1
Num Bottom Sites	1
Num Right Sites	3
Num Left Sites	3

Game concepts
Alternating
Pieces placed on Board
Step moves
Step to Empty
Adjacent Directions
Forward Directions
Leftward Direction
Rightward Direction
Played on Vertices
Reach End
Stalemate End

All concepts here: ludii.games/concepts.php?gameId=368

Other variants of FMG

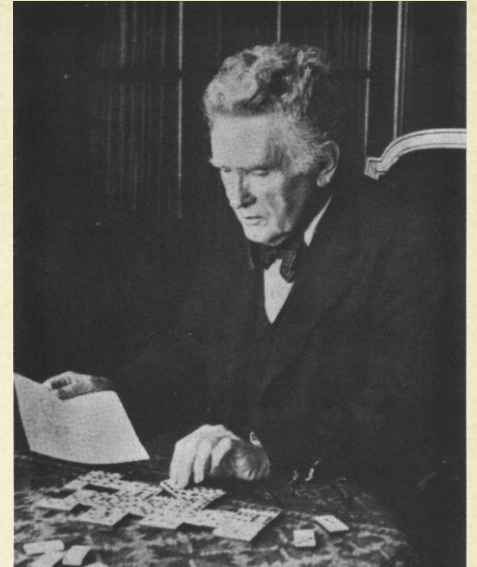
- Gardner version
- Gardner improved
- Other movements for the tower?

Ludii DEMO!

The Game of Dwarfs (or Catch the Giant!)

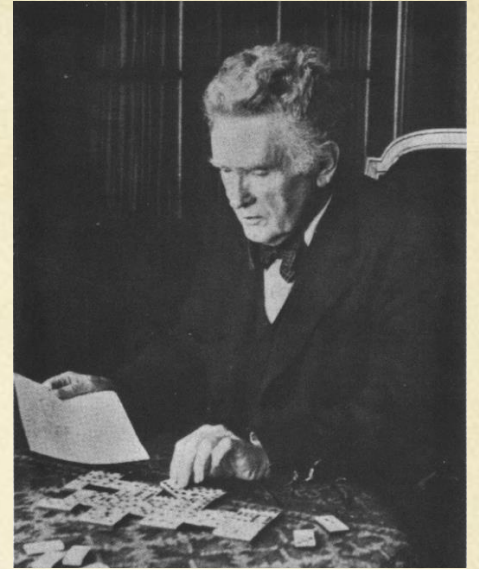
The Game of Dwarfs (or Catch the Giant!)

- 1943 : Fred Schuh, *Wonderlijke Problemen*,
Zutphen: W. J. Thieme & Cie, pp. 189-192.
Translated in 1968 in English, reissued in 2015.



The Game of Dwarfs (or Catch the Giant!)

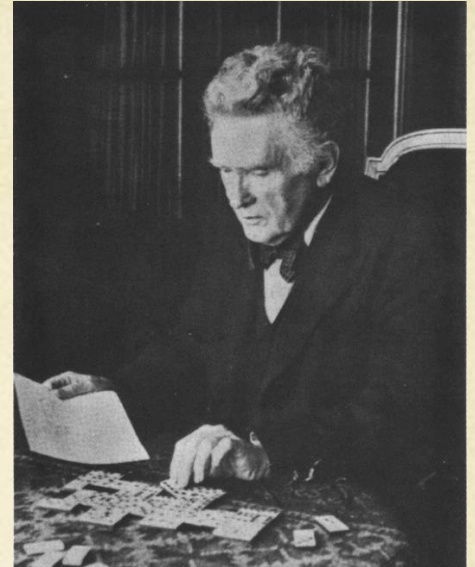
- 1943 : Fred Schuh, *Wonderlijke Problemen*, Zutphen: W. J. Thieme & Cie, pp. 189-192.
Translated in 1968 in English, reissued in 2015.
- Presents the FMG*: The Soldiers' Game



* $\triangle!$ Black can place his piece wherever he wants at the beginning + White win if Black is blocked on 0...

The Game of Dwarfs (or Catch the Giant!)

- 1943 : Fred Schuh, *Wonderlijke Problemen*, Zutphen: W. J. Thieme & Cie, pp. 189-192.
Translated in 1968 in English, reissued in 2015.
 - Presents the FMG*: The Soldiers' Game
 - The Game of Dwarfs (or Catch the Giant!)

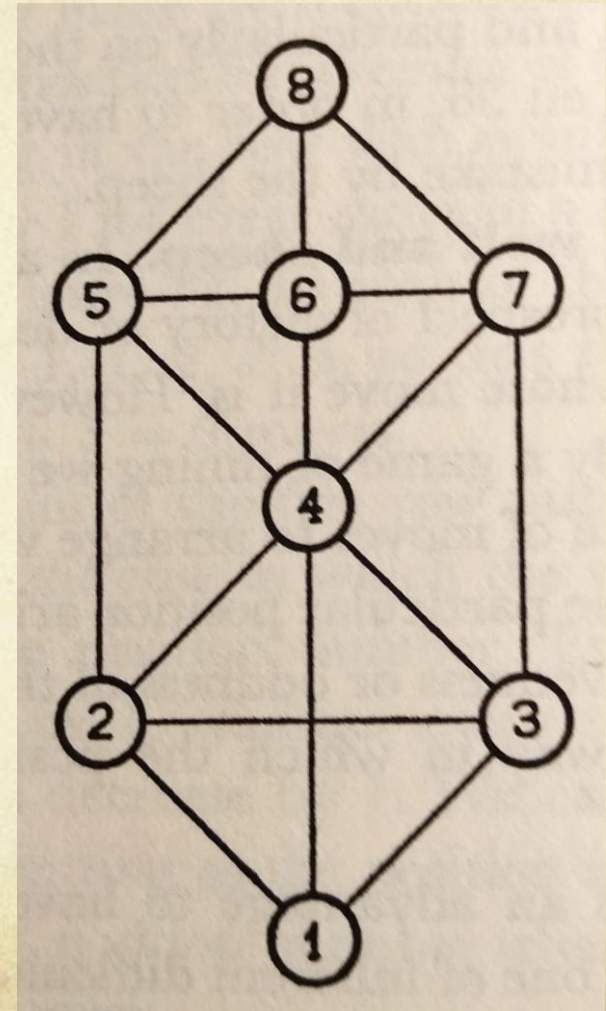


* $\triangle!$ Black can place his piece wherever he wants at the beginning + White win if Black is blocked on 0...

The Game of Dwarfs (or Catch the Giant!)

Rules

- 3 dwarfs placed on circles 1, 2, 3.
- 1 giant (P1) placed in 5, 6, 7 or 8 with a free choice.
- Aim of the giant: escape the dwarfs.
- Aim of the dwarfs: catch the giant (!) on 8.
- Moves:
 - Dwarfs can move one step only forward, diagonally and laterally (one at a time).
 - The giant can move one step in any direction.

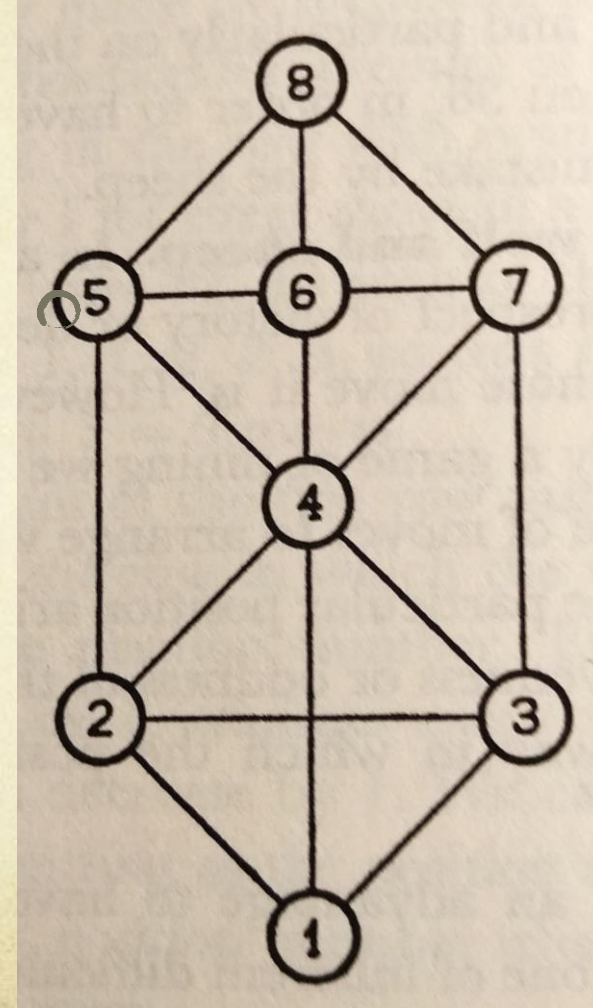
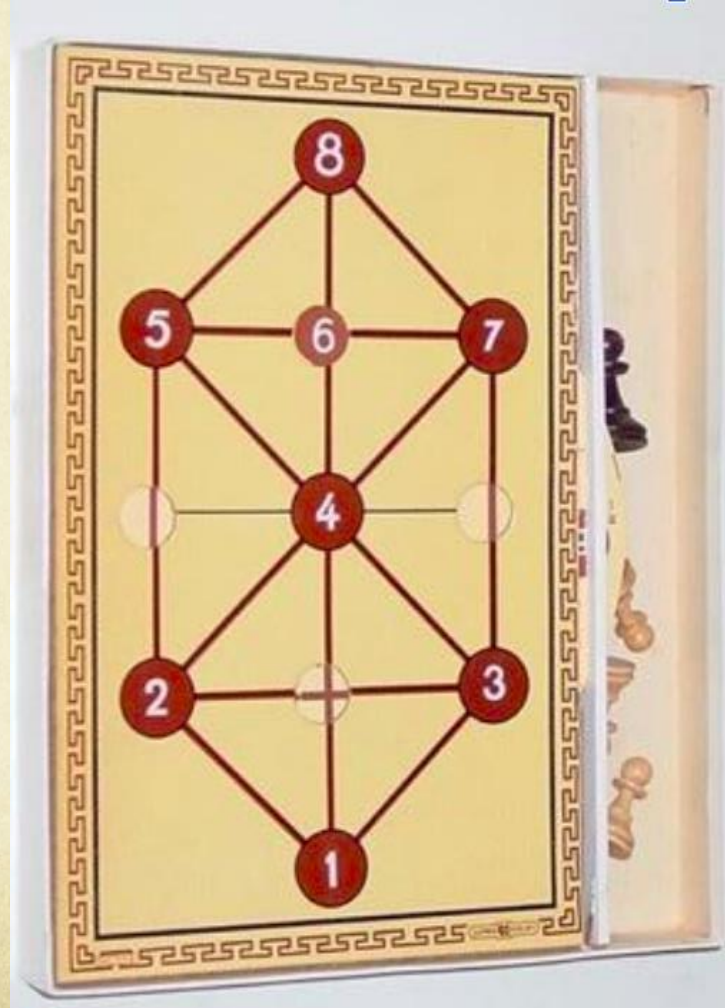


The Game of Dwarfs (or Catch the Giant!)



Commercialized in
1953.

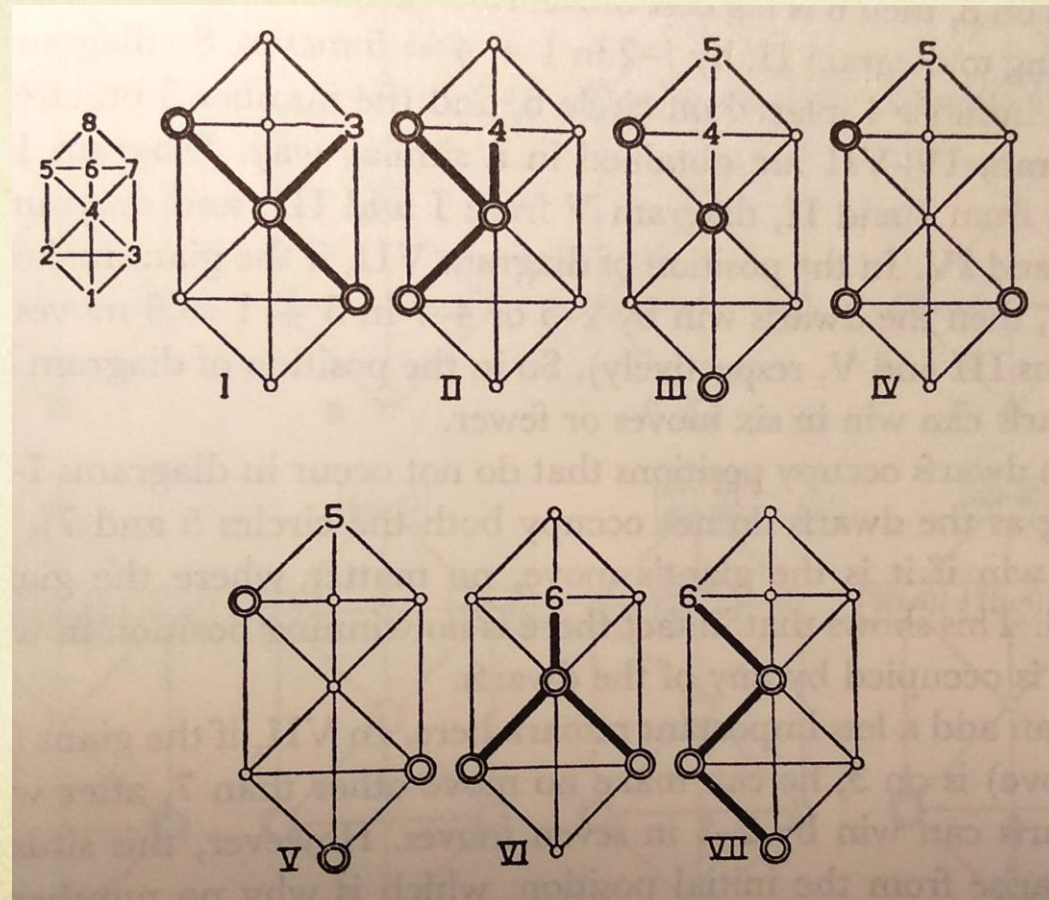
8 different boards.
14 different games.



The Game of Dwarfs (or Catch the Giant!)

Mathematical analysis of Schuh

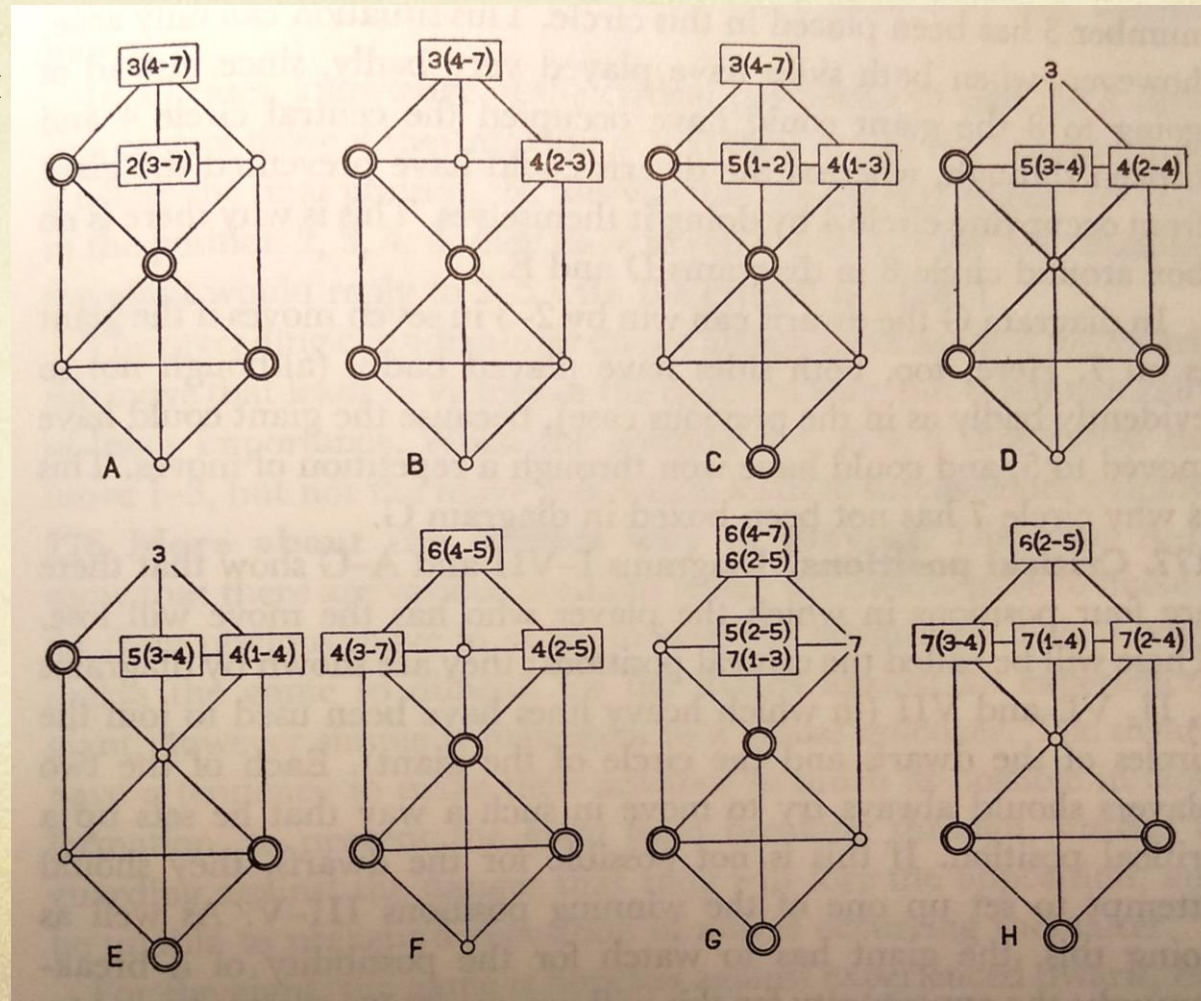
- 7 winning positions (it is the giant's turn).
- The small number indicates the number of moves in which the dwarfs can win.
- Every diagram of a higher number leads to a diagram of a smaller number.



The Game of Dwarfs (or Catch the Giant!)

Mathematical analysis of Schuh

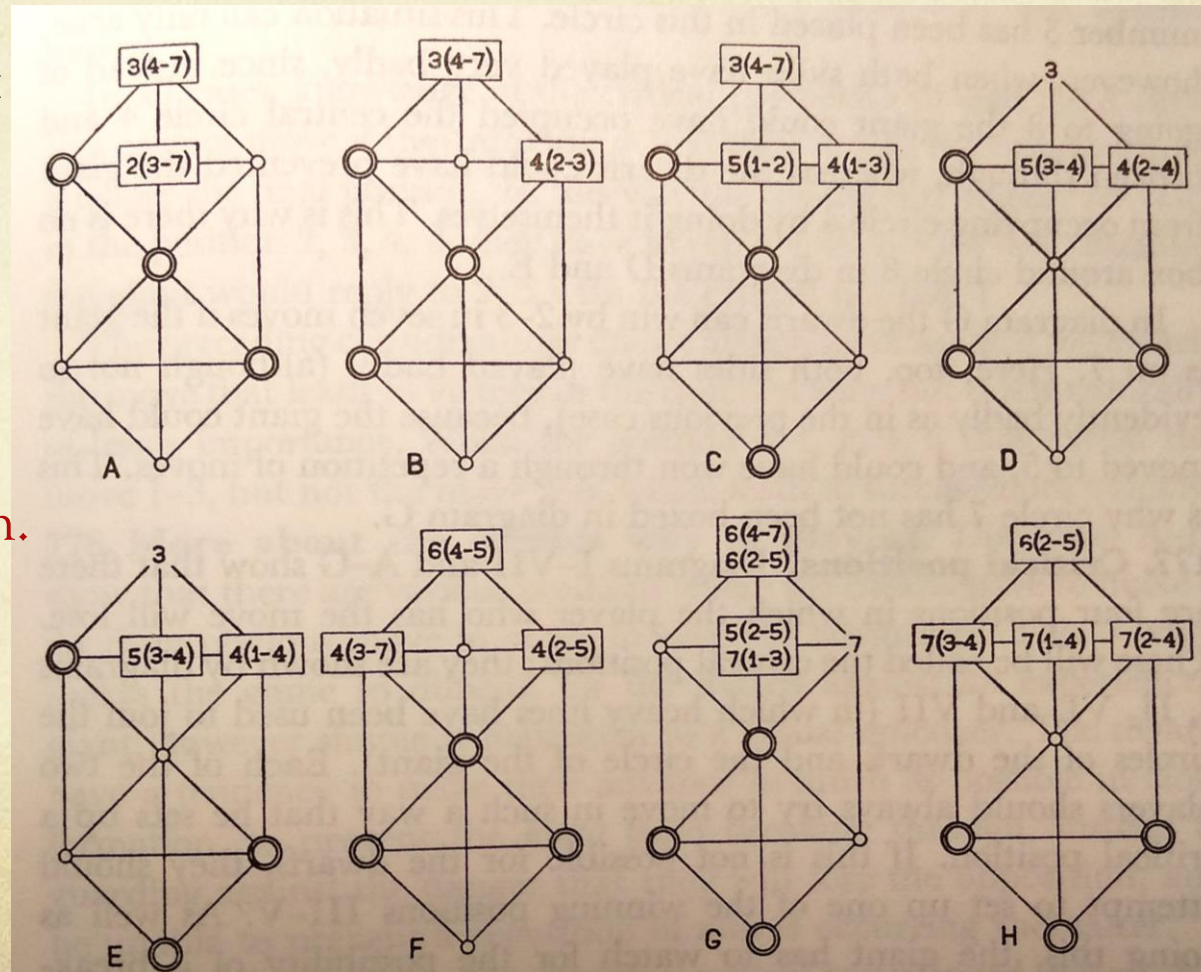
- Another way to represent where the dwarfs are to move.



The Game of Dwarfs (or Catch the Giant!)

Mathematical analysis of Schuh

- Another way to represent where the dwarfs are to move.
- Diagram H is the initial position → dwarfs can win in 7 moves maximum.



Ludii's verification by Eric ☺

Ludii DEMO!

Concluding remarks and perspectives

- Ludii = interesting tool to “check” the strategies given by the mathematicians of the past (or present)
- **Comparison of the resolution methods** (are those found by the mathematicians optimal?)
- ?? A starting point to **compare** (and connect) games by their **mathematical analysis and strategies**??
- Using this to **check reconstruction of games**? **Metrics and Concepts** computed and involved?
- Example: Thermenmuseum game (Walter’s talk, session 10).

Thank you for your attention!
Any questions?

